Assam Academy of Mathematics

MATHEMATICS OLYMPIAD

26th May, 2019

Category-II : (For Classes- VII & VIII)

Total Marks : 100 Time : 10.00 AM – 1.00 PM

1. Evaluate 6

$1^2-2^2+3^2-4^2+5^2-6^2+ \ldots\ldots + 99^2-100^2$

Soln.

$1^2-2^2+3^2-4^2+5^2-6^2+ \ldots\ldots + 99^2-100^2$

$= (1+2)(1-2) + (3+4)(3-4) + \ldots + (99+100)(99-100)$

$= -1-2-3-4-\ldots -99-100$

$= -(1+2+3+\ldots + 100)$

$= -50 \times 101$

$= -5050$

2. Simplify – 6

$$\frac{(1.2)^2 \times (0.05)^2 \div (0.25)^2}{(0.1)^2 \div (0.01)^2} \div 0.00288$$

Soln.

$$\frac{(1.2)^2 \times (0.05)^2 \div (0.25)^2}{(0.1)^2 \div (0.01)^2} \div 0.00288$$

$$= \frac{\left(\frac{1.2 \times 0.05}{0.25}\right)^2}{\left(\frac{0.1}{0.01}\right)^2} \div 0.00288$$

$$= \frac{\left(\frac{1.2 \times 0.01}{0.05}\right)^2}{10^2} \div 0.00288$$

$$= \frac{\left(\frac{1.2}{5}\right)^2}{100} \times \frac{1}{.00288}$$

$$= \frac{(.24)^2}{100} \times \frac{100000}{288}$$

$$= \frac{.24 \times .24}{100} \times \frac{100000}{288}$$

$$= \frac{\cancel{24}^{2} \times \cancel{24}}{100 \times 100} \times \frac{1000}{\cancel{288}_{\cancel{12}}}$$

$$= \frac{2}{10}$$

$$= 0.2$$

3. In the following multiplication sum, each of the digits from 1 through 9 appears exactly once in the multiplicand, multiplier and the product. One digit being known, supply the remaining digits. 7

Soln.

$$\begin{array}{r} 2\ a\ b \\ \times\quad c\ d \\ \hline e\ f\ g\ h \end{array}$$

Through trial and error, we can get the solutionas

$$\begin{array}{r} 2\ 9\ 7 \\ \times\quad 1\ 8 \\ \hline 5\ 3\ 4\ 6 \end{array}$$

4. How many 3 digit numbers are there for each of which 7 appears just once only. 6+1

Soln

The unit's place can be filled up in 10 different ways since any one of 0, 1, 2, 3, ..., 9 can be put in the units place. Whatever digit is put in unit's place the ten's place can again be filled up in 10 ways. So the unit's and ten's places can be filled in $10 \times 10 = 100$ ways.

Now, the hundred's place can be filled in 9 ways since 0 is not allowed in hundred's place.

Thus the total number of 3 digit numbers is

$9 \times 10 \times 10 = 900$.

Similarly total no. of numbers with 7 occurring in none is $9 \times 9 \times 8 = 648$

Therefore, no. of three digit numbers with 7 occuring once, twice or thrice in each = $900 - 648 = 252$

Let us count the numbers in which 7 is repeated twice or thrice.

There are 8 three digit numbers with 7 in both unit and ten's places.

There are 9 three digit numbers with 7 in unit's and hundred's places. Also there are 9 three digit numbers 7 in tens and hundred places.

Altogether there are $8+9+9 = 26$ three digit numbers with 7 repeated twice.

Finally, there is one three digit number with 7 repeated thrice.

Therefore, number of three digit numbers with 7 occuring just once is $252-26-1 = 225$

No of three digit numbers with 7 appearing only in units place is $9 \times 8 = 72$

Otherwise,

No of three digit numbers with 7 appearing only in ten's place is $9 \times 8 = 72$

No of three digit numbers with 7 appearing onlny is hundred place is $9 \times 9 = 81$

Therefore no. of three digit numbers with 7 appearing just once in each is $72+72+81 = 225$.

5. What are the two natural numbers whose difference is 66 and the least common multiple is 360. 6+1

Soln.

The HCF of two numbers will be same as the HCF of the difference and LCM of the numbers.

Now difference of the number is 66

Their LCM is 360

$66 = 6 \times 11$

$360 = 6 \times 60$

Hence, HCF of 66 and 360 in 6.

This means the HCF of the two numbers is also 6.

If a and b be the natural numbers then

$ab = HCF \times LCM = 6 \times 360 = 2160$

Now, $a-b = 66$

But $(a+b)^2 = (a-b)^2 + 4ab$

$= 66^2 + 4 \times 2160$

$= 12996$

$= 114^2$

$\therefore$ a+b = 114.

$$\therefore a = \frac{1}{2}\left[(a+b)+(a-b)\right]$$

$$= \frac{1}{2}\left[114+66\right] = \frac{1}{2}\times 180 = 90$$

$$\therefore b = 90-66 = 24$$

The numbers are 90 and 24

6. Find two unequal numbers A and B such that A+n is a factor of B+n for all values of n from 1 to 11. 7

Soln.

Consider A=1. B=1×2×3× ..... ×11×12+1

For n=1, A+n=2, B+n=1×2×.....×12+1+1=2×(3×4×...×12+1)

$\therefore$ A+n | B+n for n=1

For n=2, A+2=3, B+2=3 (1×2+4×... ×12+1)

$\therefore$ A+n | B+n for n=2

n=11, A+11=12, B+11=12×(1×2×...×11+1)

$\therefore$ A+11 | B+11 for n=11

Thus the values of A and B are

1 and 1×2×2×3...×11×12+1 respectively.

7. Find the greatest prime number that will divide 12260 leaving remainder 17. 6

Soln.

12260-17 = 12243

Now

$$\begin{array}{r|l} 3 & 1\ 2\ 2\ 4\ 3 \\ \hline 7 & 4\ 0\ 8\ 1 \\ \hline 11 & 5\ 8\ 3 \\ \hline & 5\ 3 \end{array}$$

Hence greatest prime dividing 12260 leaving remainder 17 is 53.

8. Find the largest number which would divide 50 and 60 leaving remainders 8 and 4 respectively. 6

Soln.

50–8 = 42

60–4 = 56

Now 42 = 2×3×7

56 = 2×2×2×7

Hence HCF of 42 and 56 is 2×7=14.

Hence the largest number dividing 42 and 56 leaving remainders 8 and 4 respectively is 14.

9. A student was asked to divide a number by 385. But in stead of applying long division method he applied short division method by using factors of 385 viz 5, 7 and 11 and in the process he obtained remainders 2, 4 and 10 respectively. What would be the remainder if the method of long division by 385 is applied? 6

Soln.

If q_1, q_2, q_3, be the quotients obtained by dividing the number N by 5, 7 and 11 leaving remainders 2, 4 and 10 respectively.

Then

$N = 5q_1 + 2$

$q_1 = 7\times q_2+4$ and $q_2=11q_3+10$

$\therefore N = 5\ (7q_2+4) = 35\ q_2+20$

$= 35\ (11q_3+10)+20$

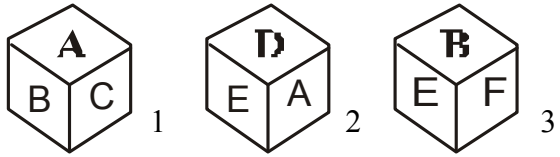
$= 385q_3+350+20$

$= 385q_3+370$

The remainder obtained by dividing the number by 385 is 370.

10. Three different views of the same cube with differently coloured faces are shown below. What is the colour of the bottom face (the face opposite to A) in figure 1? 6

Soln.



From figure 2 and figure 3, four faces adjacent to E are A, D (in fig2), B and F (as in fig-3)

Therefor the face apposite to A in fig2 is B or F. But B is adjacent to A in fig-1.

Thus the face opposite to A must be F.

11. There is a circular path around a sports field. Priya, Neha and Mina respectively take 18 minutes, 12 minutes and 8 minutes to drive one round of the field. If they start together at the same point and along the same direction, after how many minutes will they meet again at the starting point? 6

Soln.

The time required by the three runners to come together for the first time after start must be the LCM of 18, 12 and 8

Now

$$\begin{array}{r|l} 6 & 18,\ 12,\ 8 \\ \hline 2 & 3,\ 2,\ 8 \\ \hline & 3,\ 1,\ 4 \end{array}$$

Hence required time of meeting together after start is $6\times2\times3\times4=144$ minutes.

12. At what time between 7 and 8 O'clock the hour hand and minute hand will be together? 7

Soln.

At 7, the hour hand is at 7 and minute hand is at 12. Let the hour hand crosses x divisions from 7 when the minute hand overlaps with the hour hand. Then the minute hand has already crossed 35+x division.

But the ratio of divisions crossed by hour hand and minute hand is 5:60 or 1:12

$$\therefore \frac{x}{1}=\frac{35+x}{12}$$

$$\Rightarrow 12x-x=35$$

$$\Rightarrow x=\frac{35}{11}=3\frac{2}{11}$$

$\therefore$ The hour hand and minute hand will come together between 7 and 8 hour at $35+3\frac{2}{11}=38\frac{2}{11}$ minutes past 7 o'clock.

13. The cost of 4 chairs and 5 tables is Rs. 14800/- and that of 5 chairs and 4 tables is Rs. 14000/-. Find the price of a chair and of a table. 7

Soln.

Cost of 4 chairs and 5 tables is Rs. 14800

Cost of 5 chairs and 4 tables is Rs. 14000

Subtracting–

Cost of 1 table - Cost of 1 chair is 14800-14000=800

$\therefore$ Cost of 1 table = Rs. 800 + cost of 1 chair.

Then the cost of 4 chairs and 5 tables.

= cost of 4 chairs and cost of 5 chairs + 4000

= cost of 9 chairs + 4000

By condition,

Cost of 9 chairs + 4000 = 14800

$\therefore$ Cost of 9 chairs = 10800

$\therefore$ Cost 1 chair is 10800÷9

=1200

Therefore the cost of 1 table is 1200 + 800
= 2000.

14. Two trains 100 kilometers apart are moving at a speed of 10 and 15 kilometers per hour opposite to each other. If the slower train starts at 3 PM and the other starts at 2 PM, at what time will they meet together? 6

Soln.

The trains are 100 Km apart.
Let T_1 and T_2 be the trains moving at 10 Km and 15 Km per hour towards each other.
By condition T_1 starts at 3 PM while T_2 starts at 2 PM.
By the time T_1 starts moving, T_2 has already moved 15 Km towards T_1.
Therefore the trains are 100-15=85 km apart at 3 PM.
If T_1 and T_2 travel x and y km respectively when they meet together

Then $\frac{x}{10} = \frac{y}{15}$

$15x = 10y$

But $x+y = 85$

$\therefore 15x = 10\ (85-x)$

$\Rightarrow 15x+10x=850$

$\Rightarrow x = \frac{850}{25} = 34$

Then $y=85-34=51$

Hence the trains meet together after $\frac{34}{10}$ or $\frac{51}{15}$ hours after 3 PM.

In other words, the two trains will meet together in $3\frac{2}{5}$ hours after 3PM. i.e. at 6 hours 24 minutes PM.

15. Solve the following SUDOKU by inserting the numbers 1 through 9 in the blank squares such that each of these numbers appears only once in any row, column or any of the nine inner squares marked by bold lines. 10

	2	7		6			1	3
			2			9	5	4
3				8	1			6
		1			8	3	9	
	4						2	
	6	5	9			7		
6			7	1				9
7	1	8			4			
4	5			2		1	3	

Soln.

The SUDOKU is not in correct form. The correct form is –

⑤	2	7	④	6	⑨	⑧	1	3
①	⑧	⑥	2	⑦	③	9	5	4
3	⑨	④	⑤	8	1	②	⑦	6
②	⑦	1	⑥	④	8	3	9	⑤
⑨	4	③	①	⑤	⑦	⑥	2	⑧
⑧	6	5	9	③	②	7	④	①
6	③	②	7	1	⑤	④	⑧	9
7	1	8	③	⑨	4	⑤	⑥	②
4	5	⑨	⑧	2	⑥	1	3	⑦

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